

Trefoil knot and ad-hoc classification of elementary fields in the Standard Model

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Abstract

We present an arbitrary model based on the trefoil knot to construct objects of the same spectrum as that of elementary particles. It includes ‘waves’ and three identical sets of sources. Due to Lorentz invariance, ‘waves’ group into 3 types of 1, 3 and 8 objects and ‘sources’ consists of 3 identical sets of 30+2 elements, which separate into: $1 \times 1 \times 2 + 1 \times 2 \times 2 + 3 \times 2 \times 2 + 3 \times 1 \times 2 + 3 \times 1 \times 2$ and another $1 \times 1 \times 2$ group (which does not match classification of the Standard Model fields). On the other hand, there is no room in this construction for objects directly corresponding to Higgs-like degrees of freedom.

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1 Introduction

The idea of using knots for describing elementary fields is not new. It was proposed in 1867 by Lord Kelvin [1] for atoms. For about 20 years his theory was taken seriously. It eventually failed, but provided inspiration for the first extensive mathematical studies on knots, see e.g. [2]. Recently [3], it was found that knot structures form stable solutions in a model proposed in [4]. This model describes the 3+1 dynamics of a three-component vector of unit length. Such a vector field is a typical degree of freedom in the non-linear σ -model, a prototype relativistic quantum field theory.

The mathematical theory of knots [5] deals with knots constructed out of a “closed loop of rope”. The trefoil knot (fig. 1) is the simplest example of such torus-knots. In this letter, we will attempt to classify, in an ad-hoc manner, knot-like structures obtained from the trefoil knot by placing it into different positions of 3+1 – dimensional space-time. We will discuss related open knots as well, which can be obtained from a trefoil knot by the single cut (or by extending one of its loops to infinity). As no attempt will be made to define any underlying theory, which could lead for example to solutions numbered by knots, our presentation will demonstrate a substantial degree of ambiguity and arbitrariness.

Let us start with a glance at the lattice-like structure in 3+1 – dimensional space. From every junction there can be links directed into four directions: x , y , z and t . Every link can “vibrate” along three directions perpendicular to itself. This leads to $3 \times 4 = 12$ independent degrees of freedom. It is obvious that rotations belonging to the Lorentz group in a natural way provide a transformation relating vibrational degrees of freedom of x , y and z links but *not* t . This leads to the possible different classification of vibrational degrees of freedom for such links, namely $3 \times 4 = 12 = 3 + 9$. Now, let us realize that a simultaneous move of the endpoints of x , y and z links along the t direction is equivalent to an opposite shift of the central junction under consideration along the t direction. This leads to a possible separation of our degrees of freedom accordingly to the following pattern: $4 \times 3 = 12 = \bar{3} + \bar{8} + \bar{1}$. It should be noted, that the separation $9 = 1 + 8$ is much less founded than the previous one. Of course any attempt to draw a parallel between such classifications of vibrational degrees of freedom and the degrees of freedom of bosonic fields in the Standard Model of elementary particles must find healthy critics as arbitrary¹.

Now, let us move to the trefoil knot. It is an object of 3 dimensions. We can think of it as of a basically flat 2-dimensional object, as represented in our figures. There are 3 and just 3 such orthogonal planes: $x - y$, $y - z$, $z - x$ ². As a consequence all

¹ At least, the following objection is in place here: Why, if the t -link is not connected by Lorentz space-time transformations with space-like links and we put them into separate groups, do we consider vibrational modes of space links in the t -direction and space-like directions as belonging to the same group? A possible explanation is that vibration is on a smaller scale, where only topological properties such as continuity or closeness count.

² Alternatively, we can think of it as a 2-dimensional object spanned on one space and one time direction: $z - t$, $y - t$, $x - t$, or, what is equivalent, as a 3-dimensional object from any of three 3-dimensional sub-spaces: $x - y - t$, $x - z - t$ and $z - y - t$ including the time direction.

objects of our further classification will automatically appear in 3 identical copies³. We will continue now with a presentation of objects from just one of these classes.

To obtain the objects we will cut the trefoil knot (see fig. 1) and extend the loose ends into any direction of space time, irrespective of the original orientation of the knot plane⁴. The objects will group naturally, depending on the Lorentz-separated zones the loose ends happen to point at.

As the first choice we take both loose ends pointing into the time-direction. We end up with $1 \times 2 \times 2$ possibilities corresponding to links pointing into the future or the past and two possibilities of left- or right-handed knot (see fig 2). Such objects have loose ends that can be source of vibrations of the $\bar{3}$ modes and the knot as a whole can vibrate with $\bar{1}$ mode. Well, this looks very much like colour-singlet weak doublets of $(e, \nu_e)_L$ and $(\bar{e}, \bar{\nu}_e)_R$.

If we take one loose end pointing into one of the three possible space directions, another one into the time-direction of future or past, together with a choice of left- or right-handed knot, we have in total $3 \times 2 \times 2$ possibilities (see fig. 3). Such objects, owing to their free links, can be sources of vibrations of any of the $\bar{3} + \bar{8} + \bar{1}$ modes. Well, it looks very much like $(u, d)_L^{r,b,g} + (\bar{u}, \bar{d})_R^{r,b,g}$ 3-colour weak doublets of left-handed quarks and right-handed antiquarks.

If we take both loose ends pointing into the space-direction (see figs. 4 and 5) we end up with $(3 + 3) \times 1 \times 2$ possibilities⁵. Such objects, owing to their free links, can be sources of vibrations of the $0 + \bar{8} + \bar{1}$ modes. Well, it looks very much like two separate classes for $(u)_R^{r,b,g} + (\bar{u})_L^{r,b,g}$ and $(d)_R^{r,b,g} + (\bar{d})_L^{r,b,g}$, i.e. 3-colour weak singlets of right-handed up/down-quark and left-handed up/down-antiquark.

There are also two non-cut trefoil knots: left and right (see fig. 1). As they span no external links at all, they can be coupled to vibrational degree of freedom of type $\bar{1}$ only. That is why we can note them as objects of the type $1 \times 1 \times 2$. These two objects we will associate, again in a somewhat arbitrary way, with e_R and \bar{e}_L .

At this point we are left with the complete list of the Standard Model degrees of freedom, except for the, as yet not discovered experimentally, Higgs field. On the other hand, we still have $1 \times 1 \times 2$ possibilities of a knot structure when one of the loose ends points into the future and another one into the past (see fig. 6).

Should the cut-knot of fig. 6 be considered as just a kind of linear combinations of the basic states from $1 \times 2 \times 2$ class? We may also guess that these cut-knots are related to degrees of freedoms of ‘new physics’, e.g. the Higgs field. Please note, however that we have in total 3×2 , thus 6 such knots, corresponding respectively to left and right knot of three ‘families’. On the other hand, we need only 4 degrees

³ Again, healthy critic to any attempt to make a parallel with the existence of 3 families of the Standard Model is fully justified.

⁴ We can expect that the orientation of the knot loop, defining family, is irrelevant, but “true” first, second and third “family” knots are a kind of linear combinations or sort of superpositions of the basic states. One could even start to speculate on mass hierarchy and/or family mixing at this point.

⁵ We can argue that the two ends may either follow the same direction, giving 3 possibilities, or follow orthogonal directions: again $\frac{3 \cdot 2}{2} = 3$ possibilities.

of freedom for the Higgs field in the Standard Model.

Let us now sketch another construction, but basically equivalent to the previous one, where the function of loose ends and a knot loop simply get interchanged⁶. Let us go back to our 3+1 – dimensional lattice. If we assume that it is built out of infinite string-like lines, following the three x , y or z directions, crossing at junctions, then knots (as in fig. 6) can appear as kind of defects. Now we can realize that the knot loop may extend to surround neighbouring string-like lines or junctions in: (i) time direction, (ii) time and space directions, (iii) space direction, (iv) two distinct space directions, (v) no overlap at all. It is interesting to realize that in this way we end up with a spectrum of knots that coincides with the one presented above⁷ in our letter. It thus again coincides with the spectrum of elementary fermions, which are again grouped into 3 families.

The constructions presented above are by no means unique. They all rely on the simplest knot of a “closed loop of rope”, the trefoil knot, an object from the mathematical theory of knots⁸ [5].

I believe, however, that proliferating such ambiguous constructions does not make much sense without attempting to construct models for possible underlying dynamics, which could lead to predictions confrontable to the data. So far this construction cannot even be disproved.

Somehow I cannot make my mind up on whether this construction is entertainment or a sketchy hypothesis. In any case, this construction is simple, counts all observed elementary fields of the Standard Model and leaves very little room for anything else.

⁶Here loose ends number 3 possible families and “vibration” of closed loop can be a source of “waves”.

⁷ An exception is that our spurious state of the knot with one loose link pointing into the future and another one into the past, does not fit nicely into this new representation.

⁸ In fact they rely on basic, rather geometrical than topological properties of knots. The following properties are important: (i) the knot can be left-handed or right-handed; (ii) it provides plane (or direction) which can be positioned into three distinct positions; (iii) and two 4-vectors which can be positioned into 3+1 orthogonal directions; (iv) every 4-vector can ‘vibrate’ into all perpendicular to itself directions.

Acknowledgements

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Figure captions

1. Trefoil knot and its mirror image: two distinct objects.
2. Cut-trefoil knot; (i) both loose links pointing into future/past, (ii) knot and its mirror image. In total $2 \cdot 2 = 4$ distinct objects.
3. Cut-trefoil knot: (i) first loose link pointing into future/past, (ii) second loose link pointing into any of 3 space directions, (iii) knot and its mirror image. In total: $3 \cdot 2 \cdot 2 = 12$ distinct objects.
4. Cut-trefoil knot: (i) first loose link pointing into any of 3 space directions, (ii) second loose link pointing into the same direction, no new possibilities. (iii) knot and its mirror image. In total: $3 \cdot 1 \cdot 2 = 6$ distinct objects.
5. Cut-trefoil knot: (i) first loose link pointing into any of 3 space directions, (ii) second loose link pointing into a different direction, no new possibilities, because of symmetrization factor $\frac{1}{2}$, (iii) knot and its mirror image. In total: $3 \cdot 1 \cdot 2 = 6$ distinct objects.
6. Cut-trefoil knot and its mirror image; one loose end points into the future, the other into the past. We get just 2 possibilities.

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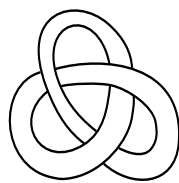
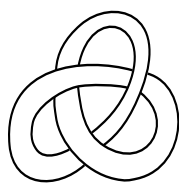


Fig. 1

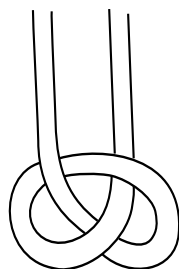
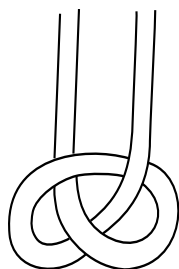


Fig. 2

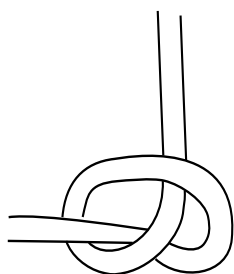
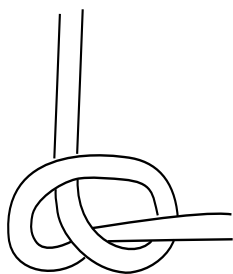


Fig. 3

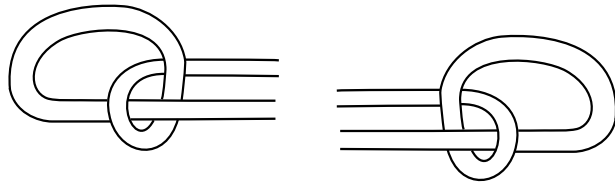


Fig. 4

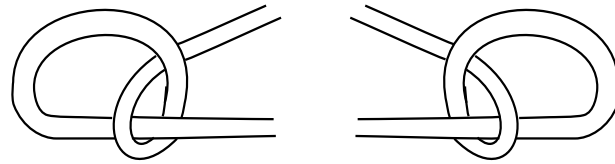


Fig.5

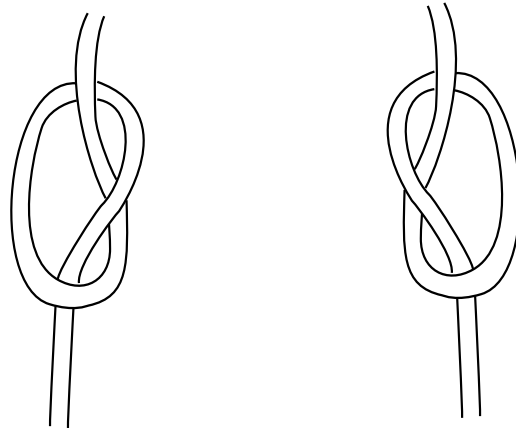


Fig. 6